

Nonlinear superconformal symmetry

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Abstract

We discuss two different nonlinear generalizations of the $osp(2|2)$ supersymmetry which arise in superconformal mechanics and fermion-monopole models.

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1 Introduction

Nonlinear supersymmetry [1, 2, 3, 4] is a supersymmetric generalization of a nonlinear symmetry characterized by a nonlinear algebra of integrals of motion. The best known examples of nonlinear symmetry are provided by the Kepler problem and by the planar anisotropic oscillator with a rational frequency ratio [5]. One of the simplest systems revealing nonlinear supersymmetry is a single-mode parabosonic oscillator system characterized by the superalgebra of the form

$$[Q_+, Q_-]_+ = P_n(H), \quad Q_\pm^2 = 0, \quad [H, Q_\pm] = 0 \quad (1.1)$$

with a polynomial function $P_n(H)$ whose order is fixed by the order of a paraboson [2]. In more general case of nonlinear supersymmetry a polynomial may include dependence on other even integrals of motion I_k ,

$$[Q_+, Q_-]_+ = P_n(H, I_k), \quad Q_\pm^2 = 0, \quad [I_k, H] = [I_k, I_{k'}] = [I_k, Q_\pm] = [H, Q_\pm] = 0, \quad (1.2)$$

playing the role of the central charges of the nonlinear superalgebra [6].

Nonlinear supersymmetry (1.2) is a nonlinear generalization of the linear $N = 2$ supersymmetry (1.1) with $P_1(H) = H$. The latter is a $(0 + 1)$ -dimensional analog of the super-Poincaré symmetry. The natural question that arises then is: does there exist a nonlinear generalization of superconformal symmetry described in $(0 + 1)$ -dimensional case by the $osp(2|2)$ superalgebra? The latter Lie superalgebra characterizes, in particular, the superconformal mechanics model symmetry [7, 8, 9].

In this contribution, based on the recent papers [10, 11, 12], we shall show that the sought-for generalization exists and, moreover, that it is a hidden symmetry of superconformal mechanics model when its boson-fermion coupling constant takes integer values. This can be compared with the nonlinear symmetry of the planar anisotropic oscillator appearing at the rational values of frequencies ratio. On the other hand, we shall see that another, more simple nonlinear version of superconformal symmetry characterizes the system of a charged fermion in the field of the Dirac magnetic monopole. The structure of the nonlinear superconformal symmetry of the fermion-monopole system is close to the form of nonlinear supersymmetry (1.2).

2 Nonlinear superconformal symmetry $osp(2|2)_n$

Let us consider the planar system of a free spin-1/2 particle described by the action

$$A = \int \mathcal{L}_0 dt, \quad \mathcal{L}_0 = \frac{1}{2} \dot{x}_i^2 - \frac{i}{2} \dot{\xi}_i \xi_i. \quad (2.1)$$

It is characterized by the nontrivial Poisson-Dirac brackets $\{x_i, p_j\} = \delta_{ij}$, $\{\xi_i, \xi_j\} = -i\delta_{ij}$, $i, j = 1, 2$, and by the integrals of motion p_i , $X_i = x_i - tp_i$ and ξ_i . Among all the functions of p_i , X_i and ξ_i , there is a set of quadratic integrals

$$H = \frac{1}{2} p_i^2, \quad K = \frac{1}{2} X_i^2, \quad D = \frac{1}{2} X_i p_i, \quad \Sigma = -\frac{i}{2} \epsilon_{jk} \xi_j \xi_k, \quad (2.2)$$

$$J = L + \Sigma, \quad (2.3)$$

$$Q_1 = p_i \xi_i, \quad Q_2 = \epsilon_{ij} p_i \xi_j, \quad S_1 = X_i \xi_i, \quad S_2 = \epsilon_{ij} X_i \xi_j, \quad (2.4)$$

where Σ is the particle's spin, and $L = \epsilon_{ij} X_i p_j$ is its angular momentum. The even, (2.2), (2.3), and the odd, (2.4), integrals form the superalgebra (only the nontrivial Poisson bracket relations are displayed)

$$\{D, H\} = H, \quad \{D, K\} = -K, \quad \{K, H\} = 2D, \quad (2.5)$$

$$\{D, Q_a\} = \frac{1}{2}Q_a, \quad \{D, S_a\} = -\frac{1}{2}S_a, \quad \{H, S_a\} = -Q_a, \quad \{K, Q_a\} = S_a, \quad (2.6)$$

$$\{\Sigma, Q_a\} = \epsilon_{ab}Q_b, \quad \{\Sigma, S_a\} = \epsilon_{ab}S_b, \quad (2.7)$$

$$\{Q_a, Q_b\} = -i\delta_{ab}2H, \quad \{S_a, S_b\} = -i\delta_{ab}2K, \quad (2.8)$$

$$\{Q_a, S_b\} = -i\delta_{ab}2D - i\epsilon_{ab}(J + \Sigma). \quad (2.9)$$

The total angular momentum J commutes with all other quadratic integrals $H, K, D, \Sigma, Q_a, S_a$, $a = 1, 2$, and the superalgebra (2.5)–(2.8) is identified as the $osp(2|2) \oplus u(1)$ with the $u(1)$ corresponding to the centre J . The Hamiltonian reduction of the system to the surface of the fixed value of J given by the constraint

$$J - \alpha = 0 \quad (2.10)$$

does not change the form of the superalgebra $osp(2|2)$, replacing J for a constant α in Eq. (2.9). Under such a reduction, the superconformal algebra generators take the form of the integrals of motion of the superconformal mechanics model [7, 8],

$$H = \frac{1}{2} (p^2 + \alpha(\alpha + 2i\psi_1\psi_2)q^{-2}), \quad (2.11)$$

$$D = \frac{1}{2}qp - tH, \quad K = \frac{1}{2}q^2 - 2tD - t^2H, \quad \Sigma = -i\psi_1\psi_2, \quad (2.12)$$

$$Q_a = p\psi_a + \frac{\alpha}{q}\epsilon_{ab}\psi_b, \quad S_a = q\psi_a - tQ_a, \quad (2.13)$$

where $q = \sqrt{x_i^2}$, $p = n_i p_i$, $\psi_1 = n_i \xi_i$, $\psi_2 = \epsilon_{ij} n_i \xi_j$, $n_i = q^{-1}x_i$, $\{q, p\} = 1$, $\{\psi_a, \psi_b\} = -i\delta_{a,b}$.

Now, let us generalize the constraint (2.10) for the constraint

$$\mathcal{J}_n - \alpha = 0, \quad \mathcal{J}_n \equiv L + n\Sigma, \quad (2.14)$$

where $n = \mathbb{N}$. The Lagrangian

$$\mathcal{L}_n = \mathcal{L}_0 - \frac{1}{2x_i^2}(\epsilon_{jk}x_j\dot{x}_k + n\Sigma - \alpha)^2 \quad (2.15)$$

with \mathcal{L}_0 given by Eq. (2.1) generates constraint (2.14) as the unique (primary) constraint for the system with the canonical Hamiltonian $H = \frac{1}{2}p_i^2$. The quantities, gauge invariant with respect to the

action of the constraint \mathcal{J}_n , are identified as observables of the system (2.15). Defining the complex variables

$$X_{\pm} = \frac{1}{\sqrt{2}}(X_1 \pm iX_2), \quad P_{\pm} = \frac{1}{\sqrt{2}}(p_1 \pm ip_2), \quad \xi_{\pm} = \frac{1}{\sqrt{2}}(\xi_1 \pm i\xi_2) \quad (2.16)$$

with nontrivial Poisson bracket relations $\{X_+, P_-\} = \{X_-, P_+\} = 1$, $\{\xi_+, \xi_-\} = -i$, one finds the quadratic observables being the integrals of motion of the system (2.15). These are the \mathcal{J}_n given by Eq. (2.14) with $L = i(X_+P_- - X_-P_+)$, and

$$H = P_+P_-, \quad K = X_+X_-, \quad D = \frac{1}{2}(X_+P_- + P_+X_-), \quad \Sigma = \xi_+\xi_-, \quad (2.17)$$

$$S_{n,l}^+ = 2^{n/2}(i)^{n-l}(P_-)^{n-l}(X_-)^l\xi_+, \quad S_{n,l}^- = 2^{n/2}(-i)^{n-l}(P_+)^{n-l}(X_+)^l\xi_- \quad (2.18)$$

with $l = 0, \dots, n$. At $n=1$ the odd observables (2.18) are the linear combinations of the odd integrals (2.4).

On the surface of the constraint (2.14), the relation

$$\mathcal{C} \equiv 4(KH - D^2) + 2n\Sigma = \alpha^2 \quad (2.19)$$

is valid, and the quantity \mathcal{C} commutes with all the set of the integrals (2.17), (2.18). The even integrals (2.17) form, as before, the Lie algebra $so(1, 2) \oplus u(1)$. Then, treating Eq. (2.14) as the constraint that fixes the orbital angular momentum L , and taking into account the relation (2.19), we find that on the surface (2.14) the integrals (2.17) and (2.18) form the nonlinear superalgebra given in addition to Eq. (2.5) by the following nontrivial Poisson bracket relations:

$$\{D, S_{n,l}^{\pm}\} = \left(\frac{n}{2} - l\right) S_{n,l}^{\pm}, \quad \{\Sigma, S_{n,l}^{\pm}\} = \mp i S_{n,l}^{\pm}, \quad (2.20)$$

$$\{H, S_{n,l}^{\pm}\} = \pm il S_{n,l-1}^{\pm}, \quad \{K, S_{n,l}^{\pm}\} = \pm i(n-l) S_{n,l+1}^{\pm}, \quad (2.21)$$

$$\begin{aligned} \{S_{n,m}^+, S_{n,l}^-\} = & -i(2H)^{n-m}(2K)^l(\alpha - 2iD)^{m-l} - i\Sigma(2H)^{n-m-1}(2K)^{l-1} \times \\ & (\alpha - 2iD)^{m-l}(n(m-l)(\alpha - 2iD) + 4\alpha l(n-m)), \quad m \geq l. \end{aligned} \quad (2.22)$$

The brackets between the odd integrals for the case $m < l$ can be obtained from (2.22) by a complex conjugation. The relations (2.5), (2.20)-(2.22) give a nonlinear generalization of the superconformal algebra $osp(2|2)$ with the Casimir element (2.19). In this nonlinear superconformal algebra, denoted in ref. [11] as $osp(2|2)_n$, the sets of odd generators $S_{n,l}^+$ and $S_{n,l}^-$, $l = 0, \dots, n$, form the two spin- $\frac{n}{2}$ representations of the bosonic Lie subalgebra $so(1, 2) \oplus u(1)$.

The quantum analogs of the $osp(2|2)_n$ generators (2.17), (2.18) are given by set of the operators [10]

$$\hat{H} = \frac{1}{2} \left(-\frac{\partial^2}{\partial q^2} + (a_n + b_n \sigma_3) \frac{1}{q^2} \right), \quad (2.23)$$

$$\hat{D} = -\frac{i}{2} \left(q \frac{\partial}{\partial q} + \frac{1}{2} \right) - \hat{H}t, \quad \hat{K} = \frac{1}{2}q^2 - 2\hat{D}t - \hat{H}t^2, \quad \hat{\Sigma} = \frac{1}{2}\sigma_3, \quad (2.24)$$

$$\hat{S}_{n,l}^+ = (q + it\mathcal{D}_{\alpha-n+1})(q + it\mathcal{D}_{\alpha-n+2}) \dots (q + it\mathcal{D}_{\alpha-n+l}) \mathcal{D}_{\alpha-n+l+1} \dots \mathcal{D}_{\alpha}\sigma_+, \quad \hat{S}_{n,l}^- = \left(\hat{S}_{n,l}^+\right)^{\dagger}, \quad (2.25)$$

where $\sigma_+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$ and

$$a_n = \alpha_n^2 + \frac{1}{4}(n^2 - 1), \quad b_n = -n\alpha_n, \quad \alpha_n = \alpha - \frac{1}{2}(n - 1), \quad \mathcal{D}_\gamma = \frac{\partial}{\partial q} + \frac{\gamma}{q}. \quad (2.26)$$

The second terms in a_n and α_n in eq. (2.26) (proportional to $(n^2 - 1)$ and $(n - 1)$) include the quantum factors \hbar^2 and \hbar , respectively, while the term $\frac{\gamma}{q}$ in \mathcal{D}_γ includes the factor $\hbar (= 1)$. These quantum corrections in the quantum analogs of the corresponding classical quantities can be obtained by the application of the reduction procedure ‘first quantize and then reduce’ to the system (2.15) [11]. This procedure fixes also the form of the quantum analogs of the classical relations (2.5), (2.20), (2.21), (2.22):

$$[\hat{H}, \hat{K}] = -2i\hat{D}, \quad [\hat{D}, \hat{H}] = i\hat{H}, \quad [\hat{D}, \hat{K}] = -i\hat{K}, \quad (2.27)$$

$$[\hat{\Sigma}, \hat{S}_{n,l}^\pm] = \pm \hat{S}_{n,l}^\pm, \quad [\hat{D}, \hat{S}_{n,l}^\pm] = i \left(\frac{n}{2} - l \right) \hat{S}_{n,l}^\pm, \quad (2.28)$$

$$[\hat{H}, \hat{S}_{n,l}^\pm] = \mp l \hat{S}_{n,l-1}^\pm, \quad [\hat{K}, \hat{S}_{n,l}^\pm] = \mp (n - l) \hat{S}_{n,l+1}^\pm, \quad (2.29)$$

$$\begin{aligned} [\hat{S}_{n,m}^+, \hat{S}_{n,l}^-]_+ = & \sum_{s=0}^{\min(l, n-m)} 2^s s! C_{n-m}^s C_l^s \times ((2\hat{K})^{l-s} (2\hat{H})^{n-m-s} \mathcal{P}_{m-l+s}(-2i\hat{D} + c_s) \Pi_+ + \\ & (-1)^{m-l} (2\hat{H})^{n-m-s} (2\hat{K})^{l-s} \mathcal{P}_{m-l+s}(2i\hat{D} + d_s) \Pi_-), \end{aligned} \quad (2.30)$$

where $\Pi_\pm = \frac{1}{2} \pm \Sigma$, $\min(a, b) = a$ (or, b) when $a \leq b$ (or, $b \leq a$), $C_l^s = \frac{l!}{s!(l-s)!}$, $\mathcal{P}_k(z)$ is a polynomial of order k ,

$$\mathcal{P}_0(z) = 1, \quad \mathcal{P}_k(z) = z(z+2) \dots (z+2(k-1)), \quad k > 0,$$

and

$$c_s = \alpha + \frac{3}{2} + n - 2(m+s), \quad d_s = -\alpha + \frac{1}{2} + 2(l-s).$$

In (2.30) it is supposed $m \geq l$, while the case corresponding to $m < l$ is obtained from it by the Hermitian conjugation. The quantum analog of the Casimir element (2.19) of the superconformal algebra $osp(2|2)_n$ takes the value

$$\hat{\mathcal{C}} \equiv 2(\hat{H}\hat{K} + \hat{K}\hat{H}) - 4\hat{D}^2 + 2n\alpha_n\hat{\Sigma} = \alpha_n^2 + \frac{1}{4}n^2 - 1. \quad (2.31)$$

The obtained nonlinear superconformal symmetry $osp(2|2)_n$ is generated by the four bosonic integrals (2.23), (2.24), which form the $so(1,2) \oplus u(1)$ Lie subalgebra, and by the $2(n+1)$ fermionic integrals (2.25) constituting the two spin- $\frac{n}{2}$ $so(1,2)$ -representations and anticommuting for the order n polynomials of the even generators. In other words, the found nonlinear generalization $osp(2|2)_n$ of the superconformal symmetry $osp(2|2)$ involves the extension of the total number of the odd generators from 4 to $2(n+1)$.

3 Hidden $osp(2|2)_n$ of superconformal mechanics model

Proceeding from the explicit form of the quantum Hamiltonian (2.23), one can find that the equality

$$\hat{H}_n^\alpha = \hat{H}_{n'}^{\alpha'} \quad (3.1)$$

takes place for the given values of n and $n' \neq n$ when the model parameter takes one of the four corresponding sets of values

$$\alpha = \alpha' = \frac{1}{2}(n + n' - 1), \quad \alpha = -(\alpha' + 1) = \frac{1}{2}(n - n' - 1), \quad (3.2)$$

$$\alpha - n = -(\alpha' + 1) = \frac{1}{2}(n + n' + 1), \quad \alpha - n = \alpha' = \frac{1}{2}(n - n' + 1). \quad (3.3)$$

The first two cases (3.2) result in the following forms of the Hamiltonian

$$\hat{H} = \frac{1}{2} \left(-\frac{d^2}{dq^2} + \frac{(n \pm n')^2 - 1}{4q^2} \mp \frac{nn'}{q^2} \Pi_+ \right), \quad (3.4)$$

where $\Pi_+ = \frac{1}{2}(1 + \sigma_3)$, and the upper and lower signs correspond to the first and second cases from (3.2). The Hamiltonian for the cases (3.3) can be obtained from Eq. (3.4) via the formal change $n' \rightarrow n' \mp 2(n + 1)$. In particular case of $n' = 1$, $n = 2k$ and $\alpha = \alpha' = k$, $k \in \mathbb{N}$, corresponding to the first relation from (3.2), the Hamiltonian takes the form

$$\hat{H} = \frac{1}{2} \left(-\frac{d^2}{dq^2} + k(k - \sigma_3)q^{-2} \right), \quad k \in \mathbb{N}. \quad (3.5)$$

It is a direct quantum analog of the classical superconformal mechanics Hamiltonian (2.11). Therefore, one can conclude that when the boson-fermion coupling constant takes integer values, $\alpha = k$, in addition to the usual superconformal symmetry of the order $n' = 1$, the quantum system (3.5) possesses also the nonlinear superconformal symmetry of the order $n = 2k$, the odd generators of which produce, via the anticommutators with the fermionic $n' = 1$ superconformal symmetry generators, the additional nontrivial bosonic integrals of motion having a nature of the half-integer degrees of the odd order polynomials of the $so(1, 2) \times u(1)$ generators, see ref. [10] for the details.

4 Fermion-monopole nonlinear superconformal symmetry

Let us consider now the system of a charged fermion in the field of the Dirac monopole described by the Hamiltonian

$$H = \frac{1}{2}P_i^2 - eB_i\mathcal{S}_i \quad (4.1)$$

with $P_i = p_i - eA_i$, $B_i = \epsilon_{ijk}\partial_j A_k = gx_i/|x|^3$, $|x| = \sqrt{x_i x_i}$, $\mathcal{S}_j = -\frac{i}{2}\epsilon_{jkl}\xi_k \xi_l$, and by the fundamental Poisson brackets $\{x_i, p_j\} = \delta_{ij}$, $\{\xi_j, \xi_k\} = -i\delta_{jk}$. The Hamiltonian (4.1) and the quantities

$$D = \frac{1}{2}X_i P_i + etB_i\mathcal{S}_i = \frac{1}{2}x_i P_i - tH, \quad (4.2)$$

$$K = \frac{1}{2}X_i^2 - et^2 B_i\mathcal{S}_i = \frac{1}{2}x_i^2 - 2tD - t^2 H, \quad (4.3)$$

where $X_i = x_i - tP_i$, together with the full angular momentum J_i , given by the relations

$$J_i = L_i - \nu n_i + \mathcal{S}_i, \quad L_i = \epsilon_{ijk} x_j P_k, \quad n_i = \frac{x_i}{|x|}, \quad \nu = eg, \quad (4.4)$$

constitute the set of the integrals of motion generating the $so(1, 2) \oplus so(3)$ symmetry [13, 14]. Since the quantization of the spin degrees of freedom gives rise to the same two-dimensional space associated with the Pauli matrices as in the superconformal mechanics model, it is rather natural to expect that the full symmetry of the fermion-monopole system has to have a nature of the superconformal symmetry. We shall show below that this is indeed so by exploiting the analogy with the two-dimensional description of the superconformal mechanics model based on the Lagrangian (2.15) $n = 1$. For the purpose, we note that the integrals of the ‘extended’ superconformal model (2.15) ($n = 1$) may be represented in a 3D form if to introduce into the system the classical odd Grassmann variable ξ_3 having the only nontrivial bracket $\{\xi_3, \xi_3\} = -i$. This will be the odd integral of motion Γ , whose quantum analog will coincide up to a numerical factor with the even quantum operator $\hat{\Sigma} = \frac{1}{2}\sigma_3$. Being interpreted as odd operator, it will anticommute with all the odd $osp(2|2)$ generators and will commute with all its even generators, and so, may be treated as a grading operator for $osp(2|2)$. Introducing the notations $x_i = (x_1, x_2, 0)$, $p_i = (p_1, p_2, 0)$, $\xi_i = (\xi_1, \xi_2, \xi_3)$ and $N_i = (0, 0, 1)$, one can understand the expressions for the $so(1, 2)$ generators from (2.2) as given by 3D scalar products, whereas the integral Σ together with the odd integrals (2.4) and $\Gamma = \xi_3$ can be represented in the 3D form as follows:

$$Q_1 = p_i \xi_i, \quad Q_2 = \epsilon_{ijk} N_i p_j \xi_k, \quad S_1 = X_i \xi_i, \quad S_2 = \epsilon_{ijk} N_i X_j \xi_k, \quad \Sigma = -\frac{i}{2} \epsilon_{ijk} N_i \xi_j \xi_k, \quad \Gamma = N_i \xi_i. \quad (4.5)$$

Then, defining for $L_j^2 \neq 0$ the vector

$$N_i = \left(1 - \frac{2\nu}{L_j^2} \mathcal{S}_k n_k\right) Y_i, \quad Y_i = L_i + \frac{2}{3} \mathcal{S}_i,$$

one can find that the quantities of the form (4.5) with p_i changed for P_i are the integrals of motion of the fermion-monopole system if the point $x_i = 0$ is excluded from its configuration space [15, 12]. Using the classical relations $\xi_i \mathcal{S}_j = \frac{1}{3} \delta_{ij} (\xi_k \mathcal{S}_k)$ and $\mathcal{S}_i \mathcal{S}_j = 0$, the integrals of motion additional to the $so(1, 2) \oplus su(2)$ generators (4.1), (4.2), (4.3), (4.4) can be represented in the form

$$Q_1 = P_i \xi_i, \quad Q_2 = \mathcal{P}_i \xi_i, \quad S_1 = X_i \xi_i, \quad S_2 = \mathcal{X}_i \xi_i, \quad \Sigma = L_i \mathcal{S}_i, \quad \Gamma = L_i \xi_i + \frac{2}{3} \mathcal{S}_i \xi_i, \quad (4.6)$$

where

$$\mathcal{P}_i = \epsilon_{ijk} L_j P_k + \frac{2}{3} \nu |x|^{-1} \mathcal{S}_i, \quad \mathcal{X}_i = \epsilon_{ijk} L_j X_k - \frac{2}{3} t \nu |x|^{-1} \mathcal{S}_i. \quad (4.7)$$

The odd integral Γ satisfies the relation

$$\{\Gamma, \Gamma\} = -i\mathcal{J}, \quad \mathcal{J} = J_i^2 - \nu^2, \quad (4.8)$$

and has zero Poisson brackets with all other even and odd integrals of motion. It is the classical analog of the Yano supercharge found in [16], see also refs. [17, 18, 19].

The even, H, K, D, Σ, J_i , and odd, $Q_a, S_a, a = 1, 2$, integrals of motion generate the superalgebra similar to (2.5)–(2.9) with the J in Eq. (2.9) changed for \mathcal{J} , and with the following relations to be different from the corresponding $osp(2|2)$ relations:

$$\{Q_2, Q_2\} = -2i\mathcal{J}H, \quad \{S_2, S_2\} = -2i\mathcal{J}K, \quad \{Q_2, S_2\} = -2i\mathcal{J}D, \quad (4.9)$$

$$\{\Sigma, Q_2\} = -\mathcal{J}Q_1, \quad \{\Sigma, S_2\} = -\mathcal{J}S_1. \quad (4.10)$$

Therefore, one concludes that classically the fermion-monopole system possesses a symmetry which is a nonlinear generalization of the superconformal symmetry $osp(2|2)$ plus decoupled rotational symmetry $su(2)$ and the supersymmetry generated by the odd supercharge Γ being effectively ‘the square root’ from the central charge \mathcal{J} . The central charge \mathcal{J} appears additively and multiplicatively in the generalized $osp(2|2)$ relations, and in this respect the nonlinear superconformal symmetry of the fermion-monopole system has a structure similar to the nonlinear superalgebraic structure (1.2).

The specific feature of the quantum analog of the described nonlinear superconformal symmetry of the fermion-monopole system is encoded in the relations

$$\hat{\Sigma} = \sqrt{\frac{\hbar}{2}}\hat{\Gamma} = \frac{\hbar}{2}(\hat{L}_i\sigma_i + \hbar), \quad \hat{\Sigma}^2 = \frac{\hbar}{4}\hat{\mathcal{J}}, \quad \hat{\mathcal{J}} = \hat{J}_i^2 - \nu^2 + \frac{\hbar^2}{4}, \quad (4.11)$$

where ν is quantized, $|\nu| = \hbar^2 k/2$, $k \in \mathbb{N}$, and we have restored the quantum constant. These relations lead to the two consequences. First, quantum mechanically the odd integral $\hat{\Gamma}$, being the grading operator of nonlinear generalization of $osp(2|2) \oplus su(2)$, is not anymore independent from the even generator $\hat{\Sigma}$ being different from it only in the constant quantum factor. Second, in representation where the squared full angular momentum operator is diagonal, $\hat{J}_i^2 = j(j+1)\hbar^2$, $j + \frac{1}{2} = |\nu| + m$, $m = 0, 1, 2, \dots$, we have $\hat{\mathcal{J}} = (|\nu| + m)^2 - \nu^2$ [14, 15]. Then, following ref. [15], one can show that in the sector $m = 0$, corresponding classically to the phase space surface given by the equations $L_i = 0$, $\mathcal{S}_j n_j = 0$, the symmetry of the system is reduced to the conformal symmetry $so(1, 2)$.

5 Discussion and outlook

To conclude, let us discuss some open problems which deserve further attention.

For the integer values of the boson-fermion coupling constant, the quantum superconformal mechanics model possesses, in addition to the superconformal symmetry described by the Lie superalgebra $osp(2|2)$, a hidden nonlinear symmetry $osp(2|2)_n$.

- It would be interesting to trace out a manifestation of this additional symmetry as well as of the nonlinearly generalized superconformal symmetry of the fermion-monopole system in the context of the corresponding quantum scattering problems.

By analogy with the anyon models, the shift of the angular momentum of the free planar fermion system may be interpreted as proceeding from the coupling of the particle to the magnetic field of the singular magnetic flux.

- Therefore, the analysis of the symmetries of the planar model of a charged particle in the field of magnetic vortex, which is closely related to the fermion-monopole system [21], could give a new perspective on the nonlinear superconformal symmetry $osp(2|2)_n$.

As in the case of nonlinear holomorphic supersymmetry [20], the nonlinear superconformal symmetry may be treated as a symmetry of a higher spin particle system [11]. Proceeding from the close similarity between the fermion-monopole system and superconformal mechanics model,

- one could expect the appearance of some generalization of the nonlinear superconformal symmetry $osp(2|2)_n$ as a symmetry for a higher spin charged particle in the field of the Dirac monopole.

Due to the observed very close similarity between the superconformal mechanics and the fermion-monopole models,

- it would also be interesting to investigate the superposition of the both systems from the point of view of the nonlinear superconformal symmetry.

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